Graphical Replication Games as a Model of Content Peering in CCNs

Valentino Pacifici and György Dán

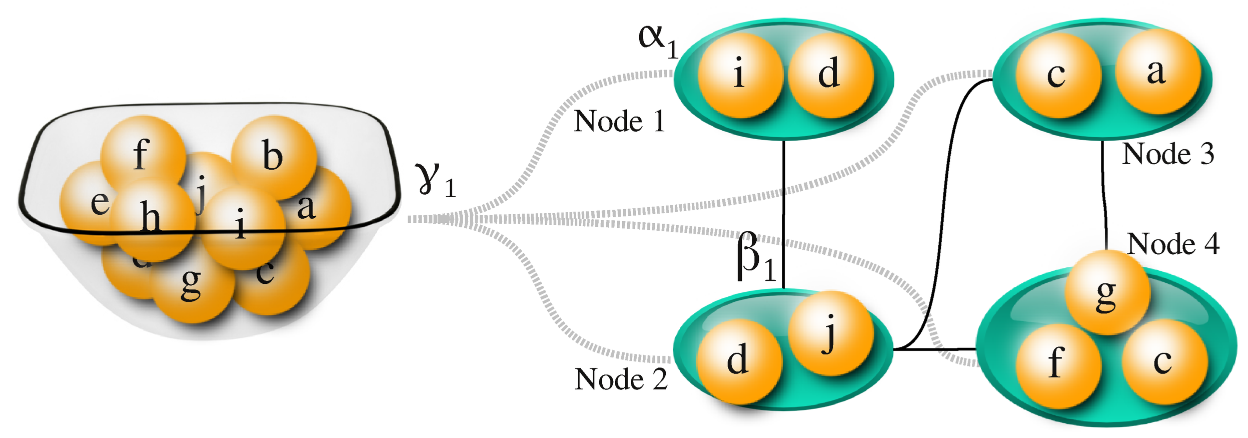
Laboratory of Communication Networks - ACCESS Linnaeus Center

KTH Royal Institute of Technology

ROYAL INSTITUTE OF TECHNOLOGY

1 Selfish Content Replication

- **The replication model**: nodes and users located near the nodes.
  1. Node $i \in N$ can replicate K_i objects from the set $\mathcal{O}$
     - $r_i = (r_i^1, \ldots, r_i^{|\mathcal{O}|})$, where $r_i^o \in \{0,1\}$ is 1 if object o is replicated in node i
     - $r_i \in \mathcal{R}_i$ is feasible if $\sum_o r_i^o \leq K_i$
  2. Limited interactions between nodes modeled with a **social graph** Γ
     - $\mathcal{N}(i)$ is the set of neighbors of node i
  3. Users generate requests for objects, the demand for object $o \in \mathcal{O}$ of the users at node $i \in N$ is $w_i^o \in \mathbb{R}_+$
  4. Cost incurred by a node to serve a user request $\begin{cases} \alpha_i & \text{if the object is replicated locally,} \\ \beta_i & \text{if the object is replicated in a neighbor,} \\ \gamma_i & \text{otherwise.} \end{cases}$



- No central authority $\Rightarrow$ No optimal solution enforced.
  - Interactions modeled as a ***Replication game*** $< N, (\mathcal{R}_i), (U_i) >$
  - **Utility function**: sum of the cost savings $U_i(r_i, r_{-i}) = \sum_o U_i^o(r_i^o, r_{-i}^o)$

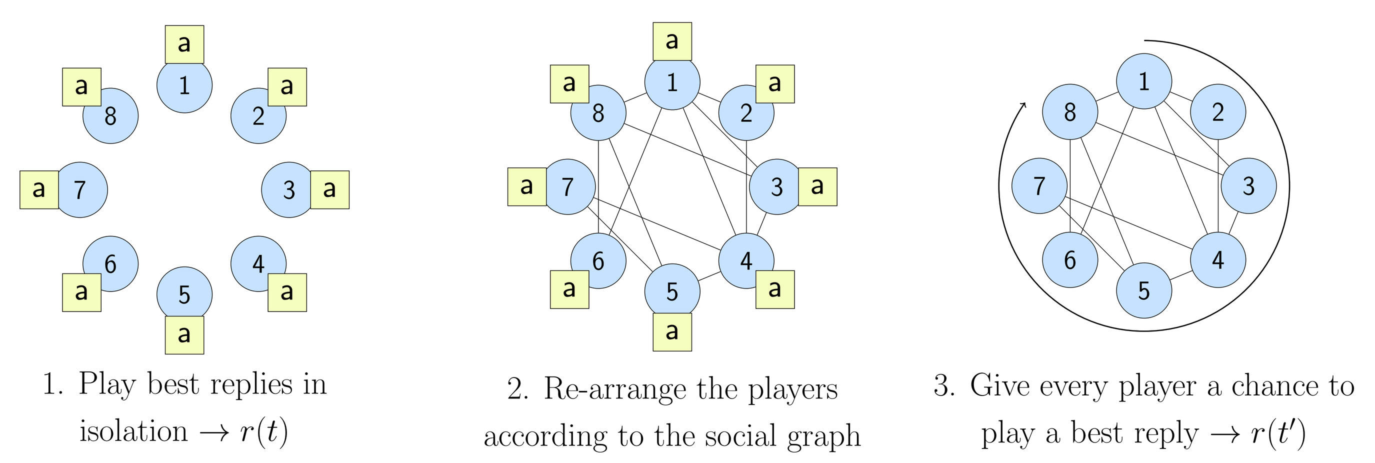
$$U_i^o(r_i^o, r_{-i}^o) = \begin{cases} 0 & \text{if } r_i^o = 0 \\ w_i^o\,[\beta_i - \alpha_i] & \text{if } r_i^o = 1 \text{ and } o \text{ is } i\text{-}busy \\ w_i^o\,[\gamma_i - \alpha_i] & \text{if } r_i^o = 1 \text{ and } o \text{ is } i\text{-}free, \end{cases}$$

  - ∗ object $o \in \mathcal{O}$ is *i-busy* if it is replicated by at least one of node i's neighbors
  - ∗ object $o \in \mathcal{O}$ is *i-free* otherwise.

- Key questions
  1. Existence of a state where every node is satisfied with the object allocation
  2. Conditions under which the nodes reach such a state updating their decisions myopically
  3. Applications in content-centric networks

2 Nash Equilibrium - Existence

- ***Nash Equilibrium***: a strategy profile r^* in which every player's strategy is a best reply to the other players' strategies.
  - $U_i(r_i^*, r_{-i}^*) \geq U_i(r_i, r_{-i}^*)\ \ \forall\, r_i \in \mathcal{R}_i,\ \forall\, i \in N.$
  - *Best reply* of player i: replication strategy that maximizes the utility of player i given the other players' strategies.
- The following algorithm always ends in a NE:



4. Play best replies in arbitrary order $\rightarrow$ NE

$\Downarrow$

Every graphical replication game possesses a pure strategy Nash equilibrium.

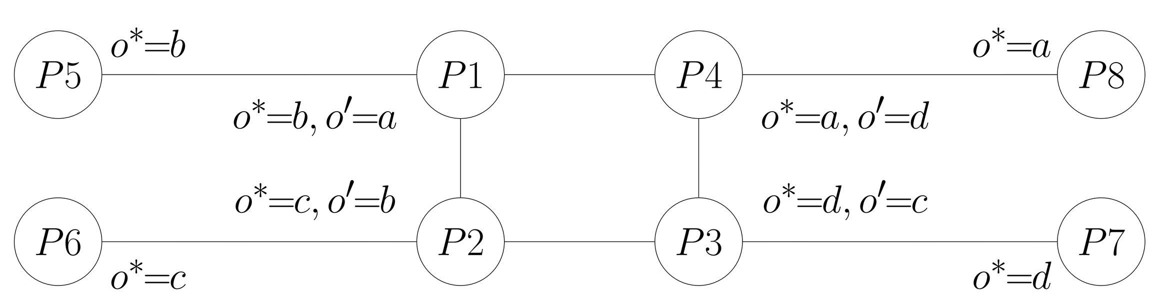
- It is possible to compute a Nash equilibrium in at most $\sum_{i \in N} \sum_{j \in \mathcal{N}(i)} K_j$ steps.

3 Nash Equilibrium - Convergence

- **Complete social graph**

  Every sequence of best replies in a replication game played over a complete social graph is finite.

- **Non-complete social graph**
  - The following graph topology allows a cycle in a sequence of best replies:



The cycle is made of the following sequence of best replies:

$$(\boldsymbol{a}, b, \boldsymbol{d}, \boldsymbol{a}) \underset{3}{\rightarrow} (\boldsymbol{a}, b, c, \boldsymbol{a}) \underset{1}{\rightarrow} (\boldsymbol{b}, \boldsymbol{b}, c, \boldsymbol{a}) \underset{4}{\rightarrow} (\boldsymbol{b}, \boldsymbol{b}, c, d) \underset{2}{\rightarrow} (\boldsymbol{b}, \boldsymbol{c}, \boldsymbol{c}, d)$$
$$\underset{1}{\rightarrow} (a, \boldsymbol{c}, \boldsymbol{c}, d) \underset{3}{\rightarrow} (a, \boldsymbol{c}, \boldsymbol{d}, \boldsymbol{d}) \underset{2}{\rightarrow} (a, b, \boldsymbol{d}, \boldsymbol{d}) \underset{4}{\rightarrow} (\boldsymbol{a}, b, \boldsymbol{d}, \boldsymbol{a})$$

4 Nash Equilibrium - Distributed Algorithm

Objective: efficient distributed algorithm for convergence to NE

Plesiochronous dynamic: player i updates her strategy at time t only if no neighboring player $j \in \mathcal{N}(i)$ updates her strategy at time t.

- From every strategy profile there exists a sequence of plesiochronous best replies that leads to a NE in a finite number of steps.

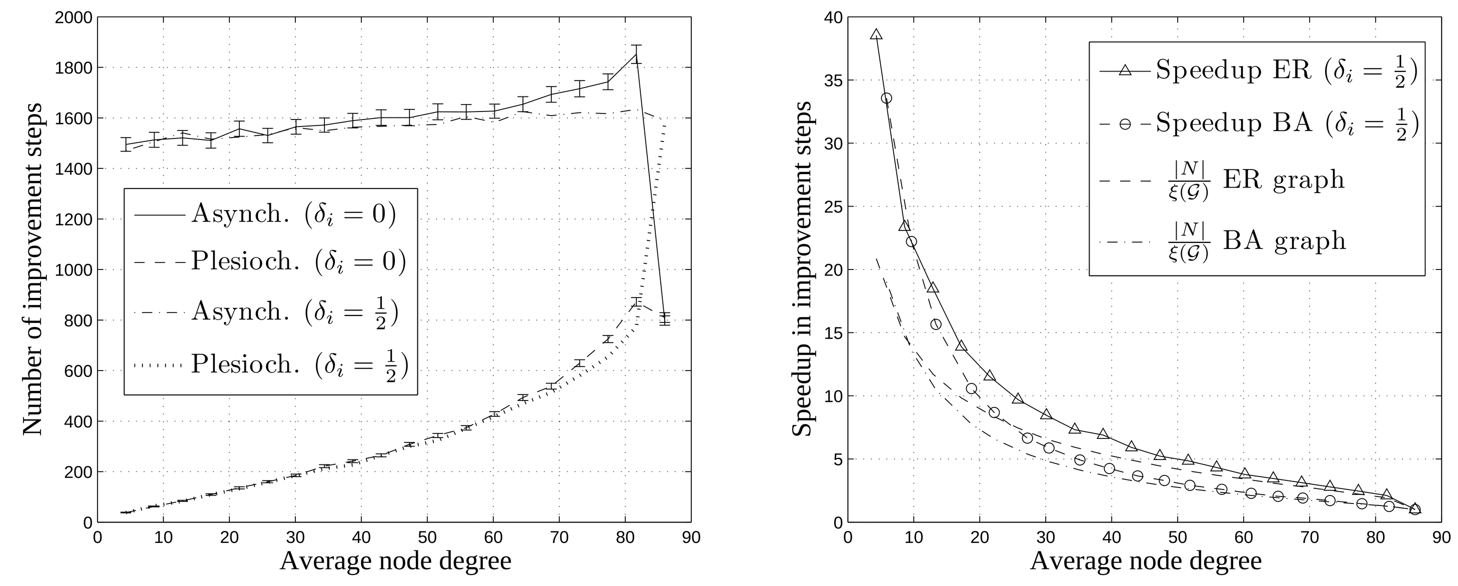
$\Downarrow$

The game is weakly acyclic under plesiochronous best replies.

- *Lazy improvement step* of player i: a strategy update of player i such that the cost saving of every inserted object exceeds that of every evicted object.
  - If $\beta_i = \alpha_i\ \forall i \in N$ and players perform exclusively lazy improvement steps,

  Every sequence of plesiochronous lazy improvement steps is finite.

- Given a coloring, the number of steps required to reach the NE is significantly smaller than for asynchronous updates



5 Autonomous Caches in a Content-centric Network

- **Scenario**: Network of Autonomous Systems
  - Each AS $i \in N$ maintains its own cache network. Summary cache r_i
  - ASs engage in **content-level peering**, modeled by the social graph Γ and $\alpha_i = \beta_i$
  - IRM assumption: request inter-arrival times are independent $\sim \mathrm{Exp}(w_i^o)$
- **Coordinated content-peering**
  - Peering ASs periodically exchange information about their cache content
  - *Cache-or-Wait algorithm*, plesiochronous updates:

    - ∗ Select a sequence of independent sets $\mathcal{I}_1, \mathcal{I}_2, \ldots$ of the social graph Γ
    - ∗ At every time slot t, allow ASs $i \in \mathcal{I}_t$ to change their cached content from $r_i(t-1)$ to $r_i(t)$
    - ∗ At the end of time slot t, AS i informs the ASs $j \in \mathcal{N}(i)$ about the new cache content $r_i(t)$

    $\downarrow$

    The previous results hold for the CoW algorithm.

  - *Cache-no-Wait algorithm*, arbitrary updates:

    - ∗ At every time slot t, allow every AS $i \in N$ to change its cached content from $r_i(t-1)$ to $r_i(t)$ performing a lazy improvement step
    - ∗ At the end of time slot t, AS i informs the ASs $j \in \mathcal{N}(i)$ about the new cache content $r_i(t)$

    $\downarrow$

    The CnW algorithm terminates in an equilibrium with probability 1.

- **Uncoordinated content-peering**
  - AS i forwards to all of its neighbors $j \in \mathcal{N}(i)$ the requests for objects o such that $r_i^o = 0$.
  - *Stable cache allocation* $\boldsymbol{r}$ for uncoordinated content-peering:

$$\forall i \in N, \forall o, p \in \mathcal{O} \quad \boldsymbol{r}_i^o = 1, \boldsymbol{r}_i^p = 0, p\ i\text{-}free \ \Rightarrow w_i^o > w_i^p$$

$\Downarrow$

Every Nash Equilibrium is a stable cache allocation for uncoordinated content peering

  - In general

    Uncoordinated peering reaches a stable cache allocation with probability 1.

  - If after a cache miss the content is instantaneously downloaded in the cache

    Uncoordinated peering reaches a stable cache allocation after a finite number of cache updates.

References

[1] V. Pacifici, G. Dán, "Convergence in Player-Specific Graphical Resource Allocation Games," *IEEE Journal on Selected Areas in Communications (JSAC)*, to appear.

[2] G. Dán, "Cache-to-cache: Could ISPs cooperate to decrease peer-to-peer content distribution costs?" *IEEE Trans. Parallel Distrib. Syst.*, vol. 22, no. 9, pp. 1469-1482, 2011.

[3] B. Chun, K. Chaudhuri, H. Wee, M. Barreno, C. Papadimitriou, and J. Kubiatowicz, "Selfish caching in distributed systems: a game-theoretic approach," in *Proc. of ACM Symposium on Principles of Distributed Computing (PODC)*, July 2004.